the representation of the, frequently bimodal, MIR phase probability distributions as unimodal.

More generally, other types of information, such as those listed in SG1 and including non-convex constraints, may be incorporated into the maps *via* ME and using the present map as a starting solution. The more information that is introduced, the smaller should be the range of allowable solutions, even when uniqueness cannot be guaranteed.

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APPENDIX 1

The algorithm used in deriving the present ME structures is essentially that outlined in our previous work (SG1; Wilkins, 1983), but with only the first constraint [equation (2)] operative. With the same notation as in our earlier works, a flow chart of the algorithm is presented in Fig. 4 and includes indications as to where additional processing (such as local smoothing) may be carried out, although such processing was not actually carried out in the present study.

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SHORT COMMUNICATIONS

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Green's tensor for a cubic crystal. By MUSTAFA DİKİCİ, *Department of Physics, İnönü University, Malatya, Turkey*

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Abstract

The equilibrium equations of classical elasticity for a cubic crystal are solved and Green's tensor for elastic displacements is derived.

1. Introduction

The point force technique provides a powerful tool for the solution of problems in the continuum theory of elasticity, particularly in the continuum theory of dislocation (Hirth & Lothe, 1968). The only difficulty arising is in representing Green's tensor. It is known explicitly for isotropy and transverse isotropy, but not general anisotropy. The aim of this paper is to obtain Green's tensor for an infinite three-dimensional body with cubic symmetry.

2. Formulation of the problem

The usual suffix notation will be employed. The summations should be carried out on repeated indices. This convention is adopted throughout this paper.

The equilibrium equation of classical elasticity is most easily obtained in a coordinate system whose bases parallel the cubic axes of the matrix. The cubic matrix has three independent elastic constants, the Voigt constants c_{11}, c_{12} and c_{44} (Hirth & Lothe, 1968). With the definitions

$$c_{12}=\lambda, \qquad c_{44}=\mu, \qquad c_{11}-c_{12}-2c_{44}=\lambda_1,$$

the fourth-order elastic tensor may be written in the following form:

$$C_{ijkm}=\lambda\delta_{ij}\delta_{km}+\mu(\delta_{ik}\delta_{jm}+\delta_{im}\delta_{jk})+\lambda_1\delta_{ij}\delta_{jk}\delta_{km}, \tag{1}$$

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where δ_{mn} is the Kronecker delta, which has the value 1 if $m=n$ and is zero for $m \neq n$.

The substitution of (1) into the equilibrium equation of classical elasticity gives

$$\mu \frac{\partial^2 u_i}{\partial x_j^2} + (\lambda+\mu)\frac{\partial}{\partial x_i}\frac{\partial u_j}{\partial x_j} + \lambda_1 \frac{\partial^2 u_i}{\partial x_i^2} + f_i = 0, \qquad (2)$$

where x_i, u_i and f_i represent the orthogonal Cartesian coordinates, the i component of the elastic displacement vector and the i component of the body force, respectively. These are the three simultaneous equations for $i = 1, 2$ and 3.

The general solution of these nonhomogeneous equations can be obtained by adding a particular solution of (2) to the general solution of the homogeneous equations obtained from (2) for $f_i = 0$.

3. General solution of the homogeneous equations

For $f_i = 0$, (2) yields the homogeneous equations. It is natural to seek a general solution of these three-dimensional equations in terms of space harmonic functions (Sokolnikoff, 1956; Love, 1927). The results of potential theory (Sokolnikoff & Redheffer, 1966) show that the divergence and curl of the displacement vector can be specified independently and that a vector $\mathbf{u}$ can be represented in terms of a scalar potential U and a vector potential $\mathbf{A}$,

$$\mathbf{u} = \nabla U + \operatorname{curl} \mathbf{A}. \qquad (3)$$

On calculating the divergence of $\mathbf{u}$ in (3), we get $\nabla \cdot \mathbf{u} = \nabla^2 U$. So that, for $f_i = 0$, (2) becomes

$$\frac{\partial^2}{\partial x_j^2}\left[\mu u_i + (\lambda+\mu)\frac{\partial U}{\partial x_i} + \lambda_1 \delta_{ij} u_i\right] = 0. \qquad (4)$$

For each i, (4) must be satisfied, so that, for each i and j,

$$\mu u_i + (\lambda+\mu)\partial U/\partial x_i + \lambda_1 \delta_{ij} u_i = \varphi_i, \qquad (5)$$

where φ_i is an arbitrary harmonic vector. The divergence of (5) gives

$$\nabla^2 U = (\lambda + 2\mu + \lambda_1 \hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j)^{-1} \partial\varphi_n/\partial x_n \qquad (n = 1, 2, 3), \quad (6)$$

where $\hat{\mathbf{e}}_i$ is a unit vector and $\hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j = \delta_{ij}$. Since φ_i is a harmonic function, $\nabla^2(x_i\varphi_i) = 2\partial\varphi_i/\partial x_i$. Thus, a particular solution of (6) is

$$U = \tfrac{1}{2} x_n \varphi_n / (\lambda + 2\mu + \lambda_1 \delta_{ij}), \qquad (7)$$

and its general solution can be obtained by adding an arbitrary harmonic function $\varphi_0' = \varphi_0/(\lambda+\mu)$ to (7). Then, the general solution of the homogeneous equations (4) is found by using (5), (7) and φ_0' in the following form:

$$(\mu + \lambda_1 \delta_{ij}) u_i = \varphi_i - \partial\varphi_0/\partial x_i - \tfrac{1}{2}(\lambda+\mu)(\lambda + 2\mu + \lambda_1\delta_{ij})^{-1} \partial(x_n\varphi_n)/\partial x_i, \quad (8)$$

involving four arbitrary harmonic functions. In connection with the two-dimensional problems, one of the space harmonics in the representation (8) can be eliminated (Sokolnikoff, 1956), so that the general solution of these three-dimensional equations involves only three independent harmonic functions.

4. A particular solution of the nonhomogeneous equations

Suppose that a point force $f_i\delta(\mathbf{r})$ is acting at the origin. For $i = 1$, the substitution of (3) into (2) yields the result

$$\frac{\partial}{\partial x_1}\left[(\lambda+2\mu)\nabla^2 U + \lambda_1 \frac{\partial^2 U}{\partial x_1^2}\right] + \frac{\partial}{\partial x_2}\left[\mu\nabla^2 A_3 + \lambda_1 \frac{\partial^2 A_3}{\partial x_1^2}\right] + \frac{\partial}{\partial x_3}\left[-\mu\nabla^2 A_2 - \lambda_1 \frac{\partial^2 A_2}{\partial x_1^2}\right] = -f_1\delta(\mathbf{r}). \qquad (9)$$

The body force can be expressed by means of formulae of the type (Love, 1927)

$$F_1 = -(f_1/4\pi)[\partial^2 r^{-1}/\partial x_1^2 + \partial^2 r^{-1}/\partial x_2^2 - \partial^2 r^{-1}/\partial x_3^2]. \quad (10)$$

From (9) and (10), one obtains

$$\left.\begin{aligned} (\lambda+2\mu)\nabla^2 U + \lambda_1 \partial^2 U/\partial x_1^2 &= (f_1/4\pi)\partial r^{-1}/\partial x_1 \\ \mu\nabla^2 A_3 + \lambda_1 \partial^2 A_3/\partial x_1^2 &= (f_1/4\pi)\partial r^{-1}/\partial x_2 \\ \mu\nabla^2 A_2 + \lambda_1 \partial^2 A_2/\partial x_1^2 &= -(f_1/4\pi)\partial r^{-1}/\partial x_3 \\ A_1 &= 0. \end{aligned}\right\} \quad (11)$$

Since $\nabla^2(\partial r/\partial x_i) = 2\partial r^{-1}/\partial x_i$, the particular solutions of (11) are

$$U = \frac{f_1}{8\pi}\frac{\partial r/\partial x_1}{(\lambda + 2\mu + \lambda_1\delta_{1j})}, \qquad A_3 = \frac{f_1}{8\pi}\frac{\partial r/\partial x_2}{(\mu + \lambda_1\delta_{1j})},$$

$$A_2 = -\frac{f_1}{8\pi}\frac{\partial r/\partial x_3}{(\mu + \lambda_1\delta_{1j})}, \qquad A_1 = 0. \qquad (12)$$

For $f_1 \neq 0$ and $f_2 = f_3 = 0$, (12) changes (9) to $\nabla^2\nabla^2|\mathbf{r}| = -8\pi\delta(\mathbf{r})$. From (3) and (12), the displacements due to f_1 are also obtained.

By repeating the same procedure for $f_2 \neq 0$, $f_1 = f_3 = 0$; and $f_3 \neq 0$, $f_1 = f_2 = 0$; and using the interchangeability of the i and j, a particular solution of (2) can be written as

$$u_{im}(\mathbf{r}) = [8\pi(\mu + \lambda_1\delta_{ij})]^{-1} \times \left[\delta_{im} f_i \nabla^2 r - \frac{(\lambda+\mu) f_m}{(\lambda + 2\mu + \lambda_1\delta_{ij})}\frac{\partial^2 r}{\partial x_i \partial x_m}\right]. \qquad (13)$$

One finds that $u_{im}(\mathbf{r})$ represents the i component of the displacement produced by a point force f_m applied in the m direction at the origin. The displacement $u_{im}(\mathbf{r})$ due to a unit point force $f_m = 1$ applied in the m direction at the origin is called the tensor Green function for the elastic displacements. It gives the response of an infinite body to a point force.

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